

Obstacles in combining the use of various tools in solving mathematical problems – why is Copy/Paste often useless

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Abstract

In several mathematical study programs there is a course, whose syllabus is primarily focused on approaches towards solving mathematical tasks with mathematical software, mostly with the programs for numerical and symbolic calculations, such as Wolfram's Mathematica, Matlab, GeoGebra.

As mathematical software is diverse and rich and each software can perform some tasks very well and others only with great efforts, quite often work with several systems, exchange the mathematical objects from one to the other and also use obtained mathematical objects in reports where text processing tools are used. Students naturally expect to do this using the copy and paste functions. However, such exchanges are often bumpy: special copy functions, plain-text adjustments, or careful verification must often be used.

The students, and we, explored transfers of mathematical objects between various programs using copy and paste. The expectations of the students and their observations are reported. Their experiments show a multitude of issues. It appears that copy and paste for regular desktop users is often useless.

Introduction

In first cycle professional study program Practical Mathematics at Faculty of mathematics and physics, University of Ljubljana, we have a course entitled Computer tools in mathematics. Study program Practical Mathematics is the first cycle professional study program. The primary goal of the program is to produce professionals capable of application of their mathematical knowledge in the working process in communications and information technology, technology, logistics, and elsewhere. There are several subject specific competences to be developed by the student:

- ability of **employment of mathematical tools at practical problem solving**,
- ability of identification of mathematical processes in professional environment,
- ability of **result analysis**,
- ability of **presentation of results**,
- ability of employment of mathematical tools in everyday environment,
- ability of initiation and conduct of software upgrade,

- ability of business process optimization,
- ability of mathematical modelling of technological processes,
- ability of employment of numerical methods at mathematical problem solving

In the preparation of the course, we focused mainly on the highlighted competences. Therefore, the main aim of this course is to show the students the practical usage of various tools in solving mathematical problems. Students will learn about and learned to use computer tools suitable in various mathematical procedures. A quite important aspect is also to learn how to report on their work.

Syllabus is primarily focused onto get used to the programs for numerical and symbolic calculations, drawing the graphs of functions, solving mathematical tasks from analysis and algebra with a computer. Tools to be considered are computer algebra systems, spreadsheets, tools for visualization and computer aided dynamic geometry systems. We agreed on the following intended learning outcomes:

- **Knowledge and understanding:** Student is familiar with computer tools and approaches to their usage in mathematical procedures. He is able to report on the process of solving a mathematical problem with computer tools.
- **Application:** The knowledge gained is used to support various subjects.
- **Reflection:** To learn about the importance of computer tools in learning and doing mathematics. To find about the importance of reporting on solving mathematical problems.
- **Transferable skills:** The subject refers to all mathematical subjects and serves for learning tools, useful in the study of these subjects.

As mathematical software is diverse and rich and each software can perform some tasks very well and others only with great efforts, we decided to use various tools, and not to focus on a single one. This included also tools to report on the content such as word processors. We envisioned a scenario where students work with several systems, exchange the mathematical objects from one to the other and use mathematical objects in reports.

This paper describes the students expectations about copy-and-pasting mathematical formulas in a learning context. For example for students it is natural to expect to use a web browser to read a document but to use a computing system to perform calculations on the formula of it. Or perhaps to take the solution provided by Mathematica and insert it into the report, prepared by Word or Libre Office Write. Students naturally expect to do this using the copy and paste functions. However, such exchanges are often bumpy: special copy functions, plain-text adjustments, or careful verification must often be used.

Having chosen different widespread software programs, students explored transfers of mathematical object between them using copy and paste. The expectations of the students and their observations are reported. The experiments show a multitude of issues. It appears that copy and paste for regular desktop users is not yet the object of mainstream testing in mainstream tools to create content.

Copy and paste from the technical point of view

When something is displayed and selectable on a computer screen, users expect it to be copy-and-pasteable: one can invoke the copy function and insert (paste) it at other places, within the same program or beyond.

Copy and paste is an exchange mechanism between different “places” inside a document. It is made of the following phases:

- An origin “place” is selected (e.g. with the mouse)
- It is copied; which transfers several copies of the content at the “place” into the system’s clipboard, one for each flavor, in order of preference.
- A target place is selected in a document being edited.
- The contents of the clipboard is pasted. The application editing that document chooses which flavor is best and inserts its associated content at the given place.

Operating systems have been offering this effect and copy-and-paste operations are part of the daily routine: for texts and images at least. For mathematical objects, this is not so. Mathematical formulas displayed within various computation or editing systems could, in principle, be transferred, but fail often to be so.

The experiment

In the first weeks of the subject Computer tools in mathematics students get used to some typical examples of mathematical software. They obtained mostly just the basic knowledge, so they know only the most common functions and functionality of the used applications. Their task was as follows. They got a requirement to solve (and to report in detail the process of obtaining the solution) a certain mathematical task. In all the tasks given the usage of different mathematical software has been foreseen. Most often the combination of computer algebra system, dynamic geometry system and numerically oriented matrix software was needed. So even when certain tasks could be solved within one environment (software) due to their limited familiarity with the software they were expected to use all of them. For reporting they mostly used the most common used word processing software like Microsoft Word or LibreOffice Writer. We were interested how they will cope with the process of exchanging the mathematical object between programs and how they will report on that. They got instructions beforehand to report on all difficulties and obstacles.

SELECTED CASES

From Word to Mathematica

We have observed a typical working path for our students. The student has a document typeset in Word. Therefore, he just uses the copy and paste functions. In Word he enters the expression using *Insert > Equation*, marks the expression, uses Ctrl-C (the copy operation), switches to Mathematica and uses Ctrl-V (Paste). How successful is he in this first part?

If using that method of copying, a warning appears when the computation is requested, which indicates that interpretation issues may appear. Unsecure users might actually decide to not press *Always* and be bothered by the warning at every paste.

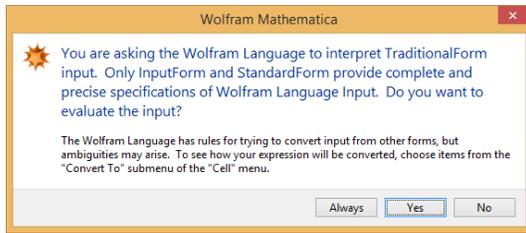


Figure 1 Mathematica's warning

Many examples perform well. The intuitive exchange just works and further computations can be done right away. Mathematica had no problems of recognizing simple expressions; it properly recognized multiple elaborate expressions depicted in Figure 2.

$$\begin{aligned} \text{In[1]:= } & \sum_{n=1}^{\infty} \left(a_n \cos \frac{n \pi x}{L} + b_n \sin \frac{n \pi x}{L} \right) \\ \text{Out[1]= } & \sum_{n=1}^{\infty} \left(\text{Cos} \left[\frac{n \pi x}{L} \right] a_n + \text{Sin} \left[\frac{n \pi x}{L} \right] b_n \right) \\ \\ \text{In[2]:= } & \frac{2}{a-b} \\ & 3 + \frac{1}{a-b} \\ \text{Out[2]= } & \frac{2}{\left(3 + \frac{1}{a-b} \right) (a-b)} \\ \\ \text{In[3]:= } & \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ \text{Out[3]= } & \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned}$$

Figure 2: Complex expressions pasted into Mathematica

Of course, there are issues when users employ notations which are beyond Mathematica's notations. Examples include 3,14159 as the first few decimals of π as done in most continental Europe countries, $\text{sen}(\alpha)$ for the sine function of alpha in the Spanish speaking countries, or the bullet for the multiplication (see Figure 3).

$$\begin{aligned} \text{In[16]:= } & 2 \bullet 3 & \text{In[29]:= } & \text{sen} (2 \pi) \\ \text{Out[16]= } & 6 \bullet & \text{Out[29]= } & 2 \pi \text{ sen} \end{aligned}$$

Figure 3: Misinterpretation of the multiplication with the bullets notation or of the sine function.

Other input modes

We assumed that students employ the menu command *Insert > Equation*. We have, however, having frequently observed that simple formulas are generally typed as part of the text and that special transformations (auto format - AF) support this practice: thus 6-b becomes 6 - b (where the minus sign has become a dash), similarly 1/4 becomes ¼. These characters stay characters when pasted into Mathematica and are thus treated it as a variable symbol as can be seen in Figure 4.

Without AF: 2 + 3 - a + b - a	In[1]= 2 + 3 - a + b - a Out[1]= 5 - 2 a + b
AutoFormat converted - (minus) to - (dash): 2 + 3 - a + b - a	In[2]= 2 + 3 - a + b - a Out[2]= 5 a (a + b) - ²

Figure 4: Mathematica treats - (in dash form) as a variable when 2+6-b+c is typed as text in Word

When using Free *form* input in Mathematica, it facilitates some of the issues described above (notably the dash instead of the minus). This mode, however, broke on other parts which worked normally, notably on all variables that are made of several letters.

1.1 From Mathematica to Word

Having worked with Mathematica, our users have some results in Mathematica's notebook and would like to copy them into Word. Many students make snapshots of Mathematica's window, crop it appropriately, and then insert into the Word as a picture. But we are interested in the process where expressions stay expressions.

If we use same Copy/Paste technique, we have not observed any problems. The expression is entered into the Word as an equation. This equation can be modified or used.

But of course the situation is not the ideal one. Namely some peculiarities of Mathematica's syntax, such as that of using square brackets for functions' arguments and function names starting with uppercase letter are not converted back. So from Word the equation

$$\sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

transferred to Mathematica and back, is converted to:

$$\sum_{n=1}^{\infty} \left(\text{Cos} \left[\frac{n\pi x}{L} \right] a_n + \text{Sin} \left[\frac{n\pi x}{L} \right] b_n \right)$$

which careful writers certainly need to adjust.

1.2 GeoGebra and Mathematica

In this experiment we are interested to what extent Mathematica and GeoGebra (in CAS mode) are capable to interchange the expressions.

We remind the reader that GeoGebra CAS's formulas are copied using the contextual menu triggered by a right click on the expressions. Only complete expressions can be copied this way.

If we use just basic operations, as can be seen in Figure 5, there are no special issues, just the fact that Copy operation in GeoGebra already equips the expression with additional parentheses and inserts the multiplication symbol.

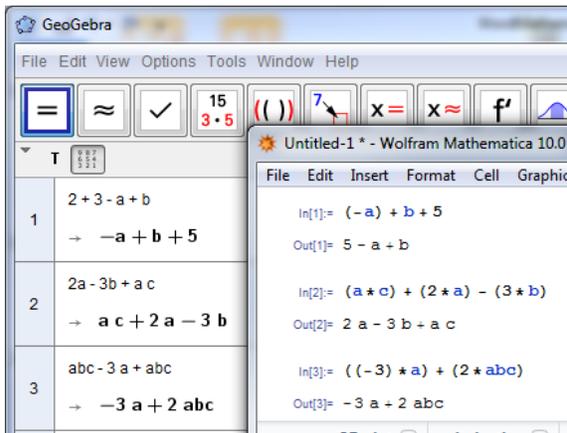


Figure 5 From GeoGebra to Mathematica: simple expressions

We cannot take just part of the output expression – only whole expression can be copied. This is not an issue when we use input expression. Here also a part of the expression can be marked and to copy it, we have to use Ctrl-C. Here GeoGebra leaves the expression intact, namely without additional parentheses.

There are also no problems with the usage of special symbols for variables, like α or β (Figure X), regardless if we copy from input or output expression.

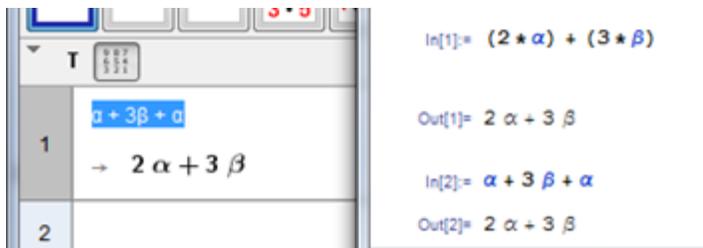


Figure X Using special symbols

Using Free Form input in Mathematica (starting the expression with = symbol) is problematic whenever we use variables with long names.

```

In[6]:= (-bla) + blo
Out[6]:= -bla + blo

In[7]:= = (-bla) + blo
          (no interpretations available) ?

+
In[9]:= = (angle + fi) - frequency
          ↶ angle fi ?
          ↷ Last close
          Row[{FinancialData["L:AGL"]}

Out[9]:= $Failed$Failed

```

Figure Y Free form input in Mathematica is problematic

As GeoGebra and Mathematica use similar notations (square brackets for commands), there are no problems whenever we have commands with the same name and same number of parameters (Expression 1 in GeoGebra vs. Expression 10 in Mathematica in the Figure 6). This works here because we copy the source expressions and the syntaxes are very similar.

This interoperability is not unbounded though. GeoGebra also has a different variant of the *Factor* command, with two parameters and a *Factors* command, which are unknown to Mathematica (compare Expressions 2, 3 and 4 in GeoGebra with Expression 11, 12 and 13 in Mathematica in Figure 6).

1	Factor[x^2 - 4] → (x - 2) (x + 2)	In[10]= Factor[x^2 - 4] Out[10]= (-2 + x) (2 + x)
2	Factor[x^2-y^2, x] → (x - y) (x + y)	In[11]= Factor[x^2 - y^2, x] Out[11]= Factor[x^2 - y^2, x]
3	Factors[x^2 - 4] → $\begin{pmatrix} x-2 & 1 \\ x+2 & 1 \end{pmatrix}$	In[12]= Factors[x^2 - 4] Out[12]= Factors[-4 + x^2]
4	Factors[2324] → $\begin{pmatrix} 2 & 2 \\ 7 & 1 \\ 83 & 1 \end{pmatrix}$	In[13]= Factors[2324] Out[13]= Factors[2324]

Figure 6 Factor command in GeoGebra and Mathematica

Using FreeForm input we were a bit more successful (Figure 5)

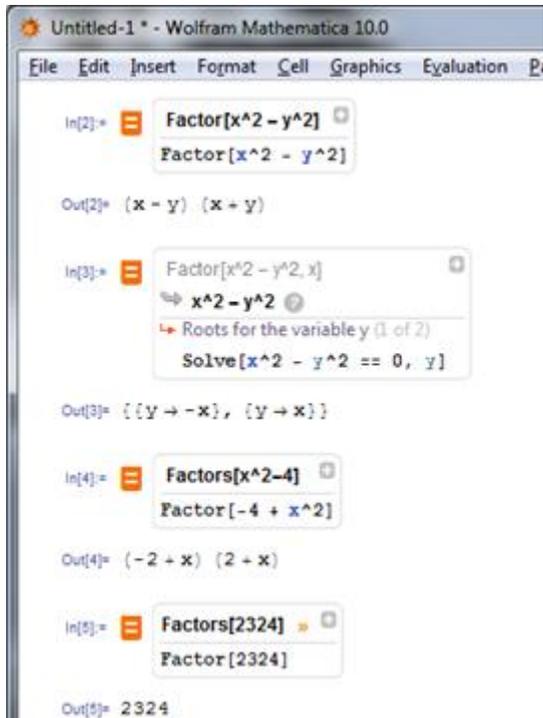


Figure 5 Using Free form input with copied Factor and Factors command from GeoGebra

Now take the opposite direction.

Ordinary expressions are not problematic, as long as we do not use special symbols. However, copying expression where we were previously successful, such as $\alpha + \beta$ fails. As can be seen in Figure 7, Mathematica inserts its own syntax (LaTeX) which creates errors in GeoGebra:

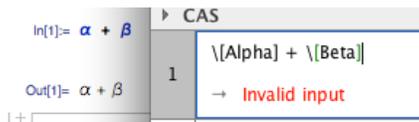


Figure 8 Copying Greek symbols from Mathematica to GeoGebra (on MacOSX)

Even use of alternative copy functions (*Copy as...*) is unsuccessful. For example, Figure 8 displays the result of pasting after having *Copy as MathML*.

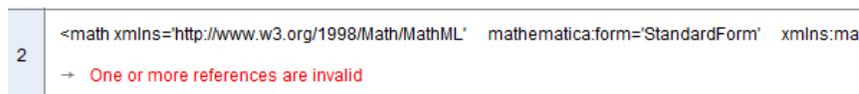


Figure 8 Using *Copy As MathML* then pasting in GeoGebra

With commands that are common to both Mathematica and GeoGebra, there are no problems – Mathematica copies them to the clipboard as “plain text”, so there is no problem with Paste. When GeoGebra does not recognize the command, it changes also parentheses from square to ordinary ones (Figure 8).

1	<code>D[x^2, x]</code> → <code>D(x², x)</code>
2	<code>FactorInteger[12345]</code> → <code>FactorInteger(12345)</code>
3	<code>Factor[x^2 - 4]</code> ○ → <code>(x - 2) (x + 2)</code>
4	<code>D[x^2 - 4, {x, 2}]</code> → <code>D(x² - 4, {x, 2})</code>

Wiris and Mathematica

Similarly to the example above, such exchanges are about two similar functions so that they complement each other's in different situations (e.g. Wiris when on browsers-only and Mathematica when at the lab, or Wiris to facilitate input and Mathematica to solve difficult differential equations...).

From Wiris to Mathematica

As noted above, Wiris puts MathML in the clipboard in text forms and not in MathML-specific flavors. This is sniffed by Mathematica which displays a dialog proposing to paste the plain text (the MathML source) or interpret MathML (Figure 9). It is probable that eager users will answer *always*, but users that expect to manipulate MathML sources might feel uncertain of doing so and thus be bothered by such a dialog every time.

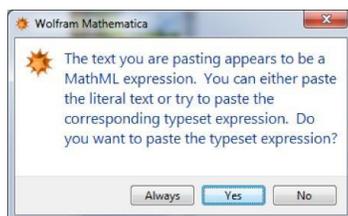


Figure 9 Mathematica's warning about MathML

After MathML is input, the same warning as in Figure 1 about the usage of Mathematica's traditional form is displayed. It seems to correspond to the interpretation of MathML-presentation into a computable form. Going beyond these warnings, and using proper notations, the exchanges work rather well as is displayed in Figure 10.

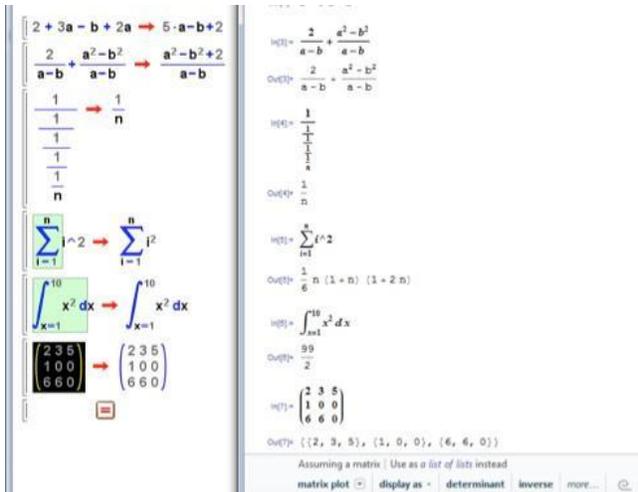
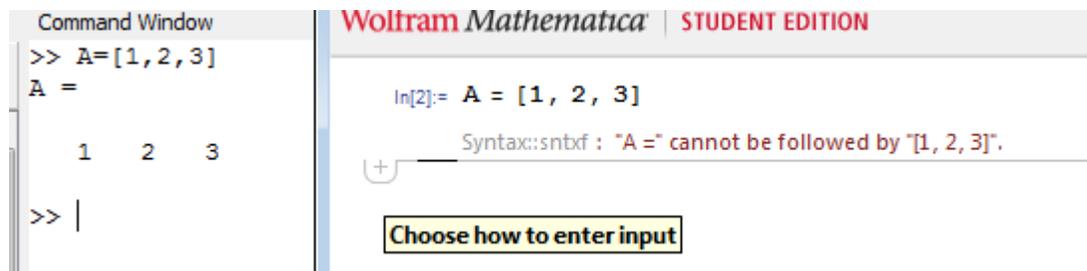


Figure 10 Copying Wiris to Mathematica

However, these exchanges failed for some mathematical symbols which are not interpreted as known notations by Mathematica. Among others, while the binomial coefficient is exchanged well; the Permutations count (without repetitions).

Matrices from Octave to Mathematica

The first problem arises when we try to define a vector:



To define a vector in octave we use square brackets, while in Mathematica we use curly brackets. The same thing happens when we try to define a matrix:



While trying to copy the defined vector or matrix from Octave to Mathematica, we get the product of each line as a result:

```

Command Window
>> A=[1,2,3]
A =
  1  2  3

>> B=[1 2 3;4 5 6; 7 8 9]
B =
  1  2  3
  4  5  6
  7  8  9

>> |

```

Wolfram Mathematica | STU

```

In[7]:= A = 1 × 2 × 3
Out[7]= 6

In[8]:= B = 1 × 2 × 3
        4 × 5 × 6
        7 × 8 × 9
Out[8]= 6

Out[9]= 120

Out[10]= 504

```

Let us check if we can copy and paste basic operators.

```

>> A+B
ans =
  5  7  9
  5  7  9
 14 16 18

>> A-B
ans =
  3  3  3
 -3 -3 -3
  0  0  0

>> A*B
ans =
  66  81  96
  30  36  42
 102 126 150

>> A'
ans =
  4  1  7
  5  2  8
  6  3  9

>> A./B
ans =
  4.00000  2.50000  2.00000
  0.25000  0.40000  0.50000
  1.00000  1.00000  1.00000

>> |

```

```

In[25]:= A + B
Out[25]= {{5, 7, 9}, {5, 7, 9}, {14, 16, 18}}

In[19]:= A - B
Out[19]= {{-3, -3, -3}, {3, 3, 3}, {0, 0, 0}}

In[20]:= A * B
Out[20]= {{4, 10, 18}, {4, 10, 18}, {49, 64, 81}}

In[21]:= A '
Out[21]= {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}'

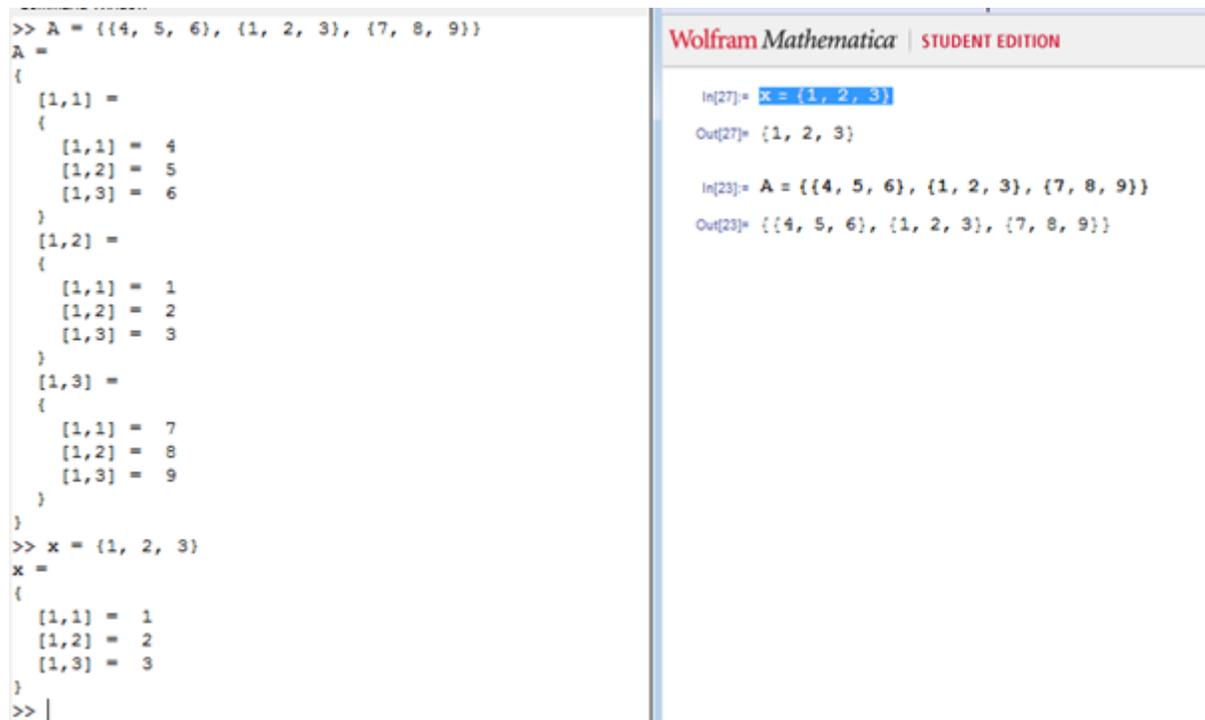
In[27]:= A ./ B
Syntax::sntxf : "A." cannot be followed by "/B".

```

We see that while addition and subtraction work, multiplication in Mathematica multiplies element by

element, while Octave does matrix multiplication. Transposing also does not work. Element by element division in Octave also cannot be copy/pasted.

When performing the transfer from Mathematica to Octave we get stacked already at the beginning, when transferring vectors and matrices.



```
>> A = {{4, 5, 6}, {1, 2, 3}, {7, 8, 9}}
A =
{
  [1,1] =
  {
    [1,1] = 4
    [1,2] = 5
    [1,3] = 6
  }
  [1,2] =
  {
    [1,1] = 1
    [1,2] = 2
    [1,3] = 3
  }
  [1,3] =
  {
    [1,1] = 7
    [1,2] = 8
    [1,3] = 9
  }
}
>> x = {1, 2, 3}
x =
{
  [1,1] = 1
  [1,2] = 2
  [1,3] = 3
}
>> |
```

```
Wolfram Mathematica | STUDENT EDITION

In[27]:= x = {1, 2, 3}
Out[27]= {1, 2, 3}

In[23]:= A = {{4, 5, 6}, {1, 2, 3}, {7, 8, 9}}
Out[23]= {{4, 5, 6}, {1, 2, 3}, {7, 8, 9}}
```

As a result we got two cell objects, which are similar to our vector and matrix, but are not what we wanted.

Conclusion

Some of observations and remarks of the students, copied from their seminar papers:

- *“... Copy/Paste method does not work with drawing plots in different programs. For drawing plots each one of used programs has its own ‘language’ which we have to use.”*
- *“When we copied expressions from Word to GeoGebra, it worked in some cases even without changing expressions. If exponents would be written with symbol ‘^’ in Word, there would not be any problems. From there on, some changes and adaptations were necessary, especially when copying expressions from GeoGebra to Mathematica”*
- *“... I was very rarely able to use Copy/Paste method to transfer expressions between programs and even then some corrections were usually needed for programs to work”*
- *“We are going to show how matrix objects can be copied from one program to another. Programs in question will be Mathematica, GeoGebra, Octave and Python (numpy). Since matrix is represented differently in each of the programs, we cannot simply just copy and paste it. Therefore, some conversion must be done.”*
- *“After several tries I gave up. The only way to transfer an expression between programs is to manually retype it ...”*

- *“To input matrix in X is surprisingly identical to input matrix in Y. But unfortunately here the joy ends.”*
- *“Apart from matrix and scalar addition and matrix scalar multiplication, the commands for matrices, even though really similar, are not compatible and are misinterpreted by those three applications.”*
- *“In this experiment we have found that Octave and Mathematica are barely compatible, so copying and pasting from one another would almost never work. They are only compatible in two basic operators + and -. Trying to copy anything from one and pasting it into another, would either generate an error or return a completely wrong answer.”*
- *“It has been an extremely tough task. I spent numerous hours doing it. I tried several techniques to force the program to properly copy/paste the expression with no success. Finally, I manually transferred the expressions. But here I had problems again as even the usage of parenthesis not even mentioning the names of the commands was not the same. I do not understand why software producers each one uses their own syntax.”*
- *“Mathematica converts expression but it works poorly.”*
- *“If we look back, we can see that matrices neither in Mathematica nor in Octave did well after Copy/Paste. But they are not the only. Also equations are not capable of adapting from Mathematica to Octave and other way around. Little less problems has calculus with variables/unknowns, while basic math operations/expressions do not have any.”*
- *“Please do not give us such tasks anymore. It is impossible. All the time switching between different syntax. At the end I did not even not if I got everything right even though the mathematics behind was luckily simple enough.”*

In the discussion afterwards they all express their deep disappointment with the software. When started the task they mostly all expected there will be just some minor problems. As one students said *“it is all mathematical software and mathematics is just one, so I expected no problems in using Copy/Paste”*

They have observed multiple issues where the created input is not allowing a transparent continuation of the work from one environment to another:

- **Loss of information at copying:** while some programmes transfer much of the information into the clipboard, allowing other programmes to recover most information in principle, quite many only put such information as the identified role of an operator or the desired spacing around them in private encodings which are difficult for other applications to receive in a consistent manner on the long term.
- **Preference for pasting plain text when other formats may be more appropriate.** It appears that most programs we have met, ignore content types that could bring more information, expecting that plain text is probably a better guess than, MathML, RTF, or HTML flavors.
- **Misinterpretation of the mathematical notation:** Employing a layout software to input mathematical formulæ leaves the user quite free to use mathematical notations in a way that is not designed to make sense. What makes sense or not, however, is not shared across the planet. Examples include the usage of the cross-sign for multiplications (which fails from Word to Mathematica). Similarly, different softwares use different function names which are not properly exchanged, e.g. the symbol P to denote the permutations’ count from Wiris to Mathematica.
- **Lack of usage of more semantic encodings:** Each of the computer algebra systems described in this paper include a rich set of mathematical operators and functions. However, any interoperability they offer to the outside world is either in linear syntax (known to be very

programme-specific) or in presentation-oriented syntax (such as MathML presentation or pictures).

- **Partial but unpredictable success:** The examples we have described above show a very diverse set of interoperability issues. There appears to be no way to explain to a user in simple terms, rules to tell when a formula that has just been processed will be transferred successfully.

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